Part 1 – A place for mathematical structure in the classroom

*Mathematical structure and structural thinking in mathematics: What are they and why they matter?*

This article is the first of three that have been written as a fulfillment of the Br John Taylor fellowship for educational research. The author’s research on this topic coincided with the completion of a Master of Research at Macquarie University, under the supervision of Professor Joanne Mulligan. The research undertaken in mathematics education looks at how to improve student engagement in mathematics through attention, to what mathematical researchers have termed “mathematical structure”.

This, the first article reports on the current research on mathematical structure and structural thinking, and how it impacts on student engagement in mathematics. The second article describes mathematical structure within the context of the current NSW K-10 mathematics syllabus, and explains the importance of developing students’ structural thinking in mathematics. The third and final article associates mathematical structure with the research on mindsets, in particular mathematical mindsets and how current practices have created fixed mindsets in the teaching and learning of mathematics, and how attention to mathematical structure increases students’ structural thinking, develops growth mindsets to support deeper understanding of mathematical procedures, and concepts to improve engagement.

**Student disengagement in mathematics**

It is well recognised that an increasing number of students are choosing General Mathematics for the NSW Higher School Certificate. This was highlighted publicly after the 2015 HSC results were released (Ting, 2015). The cause of students not attempting higher levels of mathematics and disengaging from the subject lies in earlier years before senior school calculus courses are considered.

Students begin disengaging from mathematics from early years of primary school, and this continues into junior secondary school years. In a longitudinal case study investigating student engagement in middle years of schooling, Attard (2013) collected data through interviews, focus group studies, and classroom observations and showed that positive pedagogical relationships between teacher and students were important as the foundation for students maintaining engagement in mathematical
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learning. In a previous study, Attard (2010) found that teachers without a pedagogical background in mathematics had difficulty explaining mathematical concepts to the students. With the decline in mathematics teacher graduates, the number of teachers without mathematical pedagogical experiences has increased, which has added to student disengagement. Motivational attitudes toward mathematics in early secondary school was also identified by Plenty and Heubeck (2011), they stated that students’ decisions about their perceived mathematical ability impacted on their engagement with the subject.

Enagement in mathematics is linked to how it is taught. Skemp (1976) produced his seminal paper about instrumental versus relational understanding in learning of mathematics. Skemp emphasised the need to transform mathematics teaching from an instrumental, or procedural methods, to a relational, or conceptual understanding. Kilpatrick, Swafford, & Findell (2001), and Watson & Sullivan (2008) described procedures as the ability to recall mathematical facts readily. This describes how most people remember of their mathematical experiences: rote learning facts, and procedures to be reproduced in timed tests. Sullivan (2011) stated that Skemp’s theory of relational understanding is aligned to conceptual understanding as an appreciation of mathematical ideas and relationships.

Teaching of procedures in mathematics is inherent in the teaching and learning of mathematics, as it presents an approach to complete a problem. The procedure represents the memorised method used to solve a problem, the concept being the mathematical theory, model, or idea the student needs to understand. The danger in the over use of a procedural understanding approach is that a reliance on memorising procedures without understanding the concept, or why a particular procedure is used leaves the student with a lack of understanding why a procedure is used, leaving he students unable to think deeply about the mathematical concepts presented (Mason, Stephens, and Watson 2009; Boaler, 2015b). Further to the over reliance of procedural understanding is demonstrated by research from Prescott and Cavanagh (2006) and Bobis (2000) who identified how new teachers adopted a procedural approach early in their teaching career.
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Procedural understanding, as previously mentioned, can be characterised by memorising a method. For example, to find the area of a rectangle, students are taught to multiply length by breadth, where the length and breadth are given as two separate numbers. Students, in a procedural manner, simply learn to multiply the two numbers given without understanding what area is, and why the two numbers are multiplied. This approach will yield the correct answer for the area of a rectangle, but is meaningless if the student does not recognise the length and breadth as being the adjacent sides of the rectangle. The words become pointless as many other words or symbols can be used. The multiplication process works for a rectangle but the formula cannot be applied to other plane shapes, for example a triangle. A procedural understanding requires completion of a number of similar examples, explaining the steps to get the answer, setting the students a number of similar examples to practise by repeating the process, and finally assessing the ability to repeat the process in a timed test. Procedural understanding is important in mathematics, but not as the focus. Memorising a method does not develop a deep understanding of concepts. Procedural understanding is specific to the examples given, but the process is unlikely to transfer to other situations.

Conceptual understanding requires knowledge of the basic principles of mathematics. It encourages the learners to think about the mathematics they are learning rather than recalling facts and processes. It is flexible and can be generalised to new situations.

Australian mathematics teachers have been identified as teaching principally towards a procedural understanding. In the Third International Mathematics and Science Study (TIMSS) 1999 video study, Australia was shown to have a higher proportion of nonqualified mathematics teachers, and teaching methods were dominated by a procedural approach (Lokan, McRae, & Hollingsworth, 2003). The study showed that all mathematics teachers – whether qualified or unqualified – tended towards this approach. There is a need for all teachers to be aware of the negative effects of a purely procedural approach has on learning of mathematics. The TIMSS video study identified those countries with the highest TIMSS scores tended towards more conceptual understanding in mathematics teaching.
What is mathematical structure?

Mathematical structure as focus of a report by Mason, Stephens & Watson (2009) states that teachers’ awareness of mathematical structure improves students’ engagement in mathematics. If this is the case, then there is a need to identify how the classroom teacher can do this.

As a definition of mathematical structure, Mason et al. (2009) stated it as being “the identification of general properties which are instantiated in particular situations as relationships between elements or subsets of elements of a set” (p. 10). They believed that appreciating structure is powerful in developing students’ understanding of mathematics and that attention to structure should be an essential part of mathematical teaching and learning. Mathematical structure is a precursor to structural thinking, which can be associated to cognitive structures, producing schemas that are essential in mathematical thinking and successful learning. Mason et al. (2009) stated that mathematical structure is not taught. Rather, it is an understanding of how the procedures and concepts are connected to support student learning.

In light of a significant amount of research into mathematical structure, Taylor and Wade (1965) proposed a theoretical definition as the formation and arrangement of a mathematical system within mathematical properties. Others have also referenced mathematical structure. Jones and Bush (1996) use a “building blocks” metaphor to describe mathematical structure, stating that mathematical structure is like the foundation of a building on which the content is built. They identified structural thinking in mathematics as a vehicle for helping students understand and answer the "why" questions in mathematics. In a different approach to mathematical structure, Schmidt, Houang, and Cogan (2002) were concerned with the deeper sense of mathematical structure as it connects content to deeper mathematical understanding. More recently, Mulligan and Mitchelmore (2009) identified structural thinking in preschool patterning strategies.
Mathematical structure and structural thinking in mathematics

In aligning mathematical structure to current teaching methods, Mason et al. (2009) identified mathematical structure as closely aligned to conceptual, and procedural understanding. Understanding why a procedure is used and the concept being developed helps the student to develop structural thinking awareness. Mason et al. believed that students stop learning when mathematical structure is not appreciated in the classroom. Teachers need to initiate students into mathematical structure, and cultivate it in order to mature this appreciation. By presenting research that supports this view, Mason et al. (2009) argue that students who are not encouraged to observe mathematical structure in their mathematics learning, or are not engaged in structural thinking processes, become blocked from thinking deeply about mathematics. Their point is that to develop structural thinking skills in students, teachers must have an awareness of what is mathematical structure.

In furthering their argument Mason et al. (2009) connected mastering procedures, and understanding concepts to structural thinking. They stated, that the learner would understand the relevance of the mathematics being taught, rather than relying on memorising, when the teacher’s focus is on mathematical structure. Effective mathematical thinking involves being able to use, explain, and connect mathematical properties. They use specific examples of how mathematical structure bridges the gap between procedural and conceptual understanding of mathematics in teaching and learning.

Attention to mathematical structure, as presented by Mason et al. (2009) as the overarching theory that complements and supports, procedural and conceptual understanding of mathematics should be addressed in every mathematics classroom. They provide evidence that students’ mathematical understanding is enhanced when mathematical structure is the focus of learning. To achieve this, teachers need to acknowledge mathematical structure in the content taught, the pedagogy employed, and they need to avoid relying on procedural understanding in teaching mathematics.
Mathematical structure and pedagogical content knowledge

Teacher’s understanding of mathematical structure is a significant component of pedagogical content knowledge (PCK), described by Shulman (1987) as an understanding of what to teach as well as how to teach it, is a requirement for good teaching of mathematics.

The mathematical content taught requires an awareness of mathematical structure by the teacher for effective communication to the learners. Mathematical structure enables the teacher to explain the content, so students can relate to it. The ability to demonstrate structural relationships is essential in the mathematics teacher’s pedagogy. Attention has been given to developing teacher pedagogical content knowledge (PCK) as a means of improving student learning by mathematical education researchers (Bobis, Anderson, Martin, & Way, 2011; Hill & Ball, 2004). As a component of mathematical pedagogy, mathematical structure should be included as an important part of PCK. Clarke, Clarke, and Sullivan (2012) recognised the importance of mathematical content knowledge and Vale, McAndrew, and Krishnan (2011) found that understanding both mathematical procedures and concepts improved with an awareness of mathematical structure. The knowledge of the content is essential before the concepts and procedures can be taught. Mathematical structure links these concepts, or as Mason et al. (2009) describes connects the “mythical chasm” between procedures and concepts.

Bobis (2000) did not use single out mathematical structure in her article, but identified components of mathematical structure that effective mathematics teachers understand: the interconnectedness of ideas, the ability to select, use efficient and effective strategies, challenge students to think, and encourage them to explain, listen, and solve problems. Bobis identified mathematical structure through the strategies that develop structural thinking in the students. Deeper understanding of mathematical structure will encourage mathematics teacher to use these strategies in the classroom that encourage structural thinking.
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Mathematical structure and student engagement in mathematics

Teachers of mathematics can address the issue of engagement by paying greater attention to mathematical structure in their classroom. Teachers who have an awareness of mathematical structure can develop structural thinking that engages students. (Mason et al., 2009; Mulligan & Mitchelmore, 2009; Taylor & Wade, 1965).

In their study about improving participation rates in mathematics, Brown, Brown, and Bibby (2008) surveyed over 1,500 students in 17 schools. Results from a questionnaire found that the perceived level of difficulty of the mathematics and personal lack of confidence were reasons for students not continuing with mathematics. These factors, along with a dislike and boredom, as well as a perceived lack of relevance, were also related to students’ decision not to study mathematics at senior secondary school level.

Mathematical structure aims to increase student engagement. The ability to think structurally, Mason et al (2009) asserted, gives students intrinsic reward from their enjoyment in mathematics. It is not about the mark on a test or being the fastest to answer the question. Further, Mason et al. (2009) concluded that a teacher’s awareness of structural relationships would transform students’ mathematical thinking and their disposition to engage.

Teachers understanding of mathematical structure

Various mathematics education researchers have proposed individual definitions of mathematical structure that have similarities to a broad concept, but display individual distinctions (Barnard, 1996; Jones & Bush, 1996). Others have attempted to identify how mathematical structure and structural thinking impact on students’ mathematical understanding (Jones & Bush, 1996; Mason et al., 2009; Mulligan & Mitchelmore, 2009; Vale et al., 2009). Despite this large body of research about mathematical structure, there is a lack of research about teachers’ understanding of mathematical structure and how they teach with reference to mathematical structure in junior secondary schools.
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To improve engagement in mathematics, students must become confident learners, to do this, all mathematics teachers, qualified and nonqualified need to consider mathematical structure in their pedagogical practice. The necessity for teachers to put this at the forefront of their teaching does not require a change from their current teaching habits. It simply requires a greater awareness of how students learn mathematics, and how they think when learning mathematics. Through mathematical structure, students develop structural thinking skills, which will develop a greater sense of connection to the mathematical content being taught. Once connection is made then an understanding of the procedures used and the mathematical concepts being developed will support learning. Students no longer need to rely upon memorising facts and formulae, which, are of little benefit for long-term mathematical development.

Current pedagogical practices have failed our students. The challenge for teachers, school executive, and parents is to put aside traditional methods and change the mindset that teaching mathematics for memory to complete timed tests, where a pass viewed as a determinant of those who can and those who can’t. The message this sends to young student creates a mindset of not good enough. Through the work on mindsets of Carol Dweck (2006) and Jo Boaler (2015b), as will be discussed in further detail of part 3 of this report, we now know that all people are capable of mathematical thinking. Boaler (2015a) describes the importance of making mistakes and how the brain activity when mistakes are made and corrected. Memorising does not improve brain development. Boaler describes how mathematical thinking can be achieved, and under the right conditions students’ confidence, as mathematical learners, will increase. Teachers’ awareness of mathematical structure, the development of structural thinking, and an attention to these mindsets will be the first and important step in this process.

Conclusion
The overemphasis of procedural methods in the mathematics classroom has left students disengaged in mathematics. Deeper thinking of mathematics is being achieved with a richer conceptual understanding of the content. This can be achieved by teachers’ developing an awareness of mathematical structure so students can
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understand the procedures and concepts and be able to think structurally about the mathematics, develop greater insight and appreciation in the mathematics to be learnt.

REFERENCES


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