Part 2 – A place for mathematical structure in the classroom

This is the second of three reports written to complete the Br John Taylor fellowship requirements. The theme of these reports is that of mathematical structure, this report combines research in this field, and where it applies to the current NSW K-10 mathematics syllabus through working mathematically. The author completed this research as a requirement for the Master of Education degree at Macquarie University under the supervision of Professor Joanne Mulligan.

Focus on Mathematical structure in teaching and learning

Mathematical structure is not a term commonly used by teachers in a mathematics classroom environment, but it does have a long history of use by mathematics education researchers. The notion of mathematical structure was identified by Mason, Stephens and Watson (2009) as existing as far back as Euclid, but it might be assumed that mathematical structure has been a part of mathematics since mankind started to think mathematically.

While not identifying with the term mathematical structure, Skemp (1976) made a distinction between what he referred to as instrumental and relational understandings of mathematical learning. He explained that instrumental understanding was learning a number of fixed plans with starting points and finishing points and explanations of what to do along the way, whereas relational understanding involved building up a conceptual structure or a schema that offered an unlimited number of starting points toward any finishing point, with multiple paths to get there. These terms are identified within the psychology of mathematics education literature through the work of Fischbein and Muzicant (2002, p. 248), and they represent different approaches about how mathematics is taught. The delineation between procedural and conceptual learning and understanding is bridged by mathematical structure, so that deeper thinking about the mathematics being taught is achieved. Mason et al. (2009) maintains that mathematical structure covers understanding of both procedures and concepts.

Starting from a premise, that the mastering a procedure is important when taking advantage of learning opportunities to make mathematical sense, Mason et al. (2009) stated that it is of little value to the learner if it remains a procedure. Procedural
learning simply places a burden on the learner to remember, but when procedures are associated with some appreciation of mathematical structure the learning shifts from memorising to understanding the concepts. The negative effects of procedural learning, identified by Richland, Sigler, and Holyoak (2012), are that it leaves the learner ineffective at any mathematical reasoning. It is structural thinking that allows learners to have confidence in manipulating the procedures taught, and apply the concepts to mathematical problems. Using the expression “conceptual structure”, Richland et al. (2012) acknowledged that this process allowed learners to make predictions regarding how procedures relate to the solutions, and develop new understandings about the concepts. In their study, Richland et al. (2012) examined mathematics knowledge of students who had completed the K–12 mathematics sequence and found these students were unlikely to have flexible reasoning in mathematics. Students in this study saw mathematics as a collection of procedures, rules, and facts to be remembered, and found that this became increasingly difficult as they progressed through the curriculum.

Mathematical structure foundations are in the connections between procedural understanding, which is the doing of a problem, and conceptual understanding, which is described as the knowing or understanding of why a particular procedure is used. In their article, Richland et al. (2012) proposed that students’ long-term ability to transfer and engage in mathematical knowledge is achieved through mathematics instruction that focuses on making connections between the mathematics learnt as procedure or concept.

**Teachers’ awareness of mathematical structure**

Research about mathematics teachers’ awareness of structure is related about pedagogical approaches and teacher professional learning. In a study conducted in Australia, Vale, McAndrew, and Krishnan (2011) examined out-of-field (i.e. nonqualified) mathematics teachers, after they completed a professional development course for junior secondary mathematics teachers on mathematics syllabus content and pedagogy. The researchers explored teachers’ understanding of mathematical connections and their appreciation of mathematical structure. An appreciation and awareness of mathematical structure was identified through teachers’ recognition of
mathematical relationships and properties. These teachers were then able to make connections between these relationships and properties, which they implemented in their teaching to promote student structural thinking.

The study by Vale et al. (2011) demonstrates that when teachers were taking the position of the learner, their appreciation of how structure makes connections with more complex concepts. A benefit that teachers experience when taking the role of the learner is to understand how students’ think when working on mathematical problems. This experience helps when creating pedagogical practices that support the development of structural thinking.

The teacher when aware of mathematical structure can identify connections between previous, current, and future learning of procedures and concepts. The expression “knowledge at the mathematical horizon” (Vale et al., 2011, p. 169) is used to describe how teachers’ mathematical knowledge is required to enhance students’ future mathematical learning. A definition of mathematical structure as “building blocks” (Jones & Bush, 1996) would adequately describe this idea. All mathematics learnt forms the foundation for future mathematics to be learnt.

An important outcome from the Vale et al. (2011) research was the impact that the teachers involved experienced. They felt that their deepened awareness of mathematical structure, and their ability to explore structure had increased their desire to develop a deeper understanding of pedagogical knowledge for classroom practice. In addition to this, the teachers understood that an emphasis on procedural understanding limits students in their understanding of mathematics.

Mathematical structure in mathematics curricula

Mathematical structure can be found in current international mathematics curriculum documents. The American mathematics curriculum document Common Core State Standards for Mathematics Initiative “Common Core” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) includes “look for and make use of structure” as one of the eight standards of mathematical practices used to demonstrate what students are doing when they learn...
mathematics. It recognises mathematical structure in student learning and understanding of mathematics.

Mathematical structure can be identified in the Australian Curriculum—Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2015) through the four proficiency strands of understanding, fluency, problem-solving, and reasoning. These proficiency strands reflect the multidimensional aspects of mathematical structure, and support how the content is taught through the thinking, and doing of mathematics. Essentially, these proficiency strands can be aligned with the development of structural thinking skills. In the Australian curriculum, the lack of use of the term does not mean that the concept of mathematical structure is not.

In the NSW mathematics syllabus for the Australian curriculum (NSW Board of Studies, 2012), the proficiency strands of the Australian curriculum are re-worked as working mathematically. Mathematical structure can be identified in working mathematically through the communicating, problem solving, reasoning, understanding, and fluency components. Working mathematically can be associated with students’ behaviour and affective learning patterns. In the NSW Assessment, Certification and Examination manual (Board of Studies NSW, 2016) values and attitudes exist as the affective learning or emotive aspect when doing mathematics.

Teachers’ use of mathematical structure revealed in working mathematically

In a survey, 39 mathematics teachers were interviewed by Cavanagh (2006) to examine the extent they used working mathematically in their teaching. Although this study was concerned with the NSW mathematics syllabus before the introduction of the Australian curriculum, the results are relevant. A small number of teachers interviewed were able to describe what working mathematically involved and applied it to their teaching. The majority of teachers had a very limited understanding of what working mathematically meant. As working mathematically can be aligned to mathematical structure, it appears that teachers’ lack of working mathematically awareness could transfer to a similar lack of awareness of mathematical structure.
Part 2 – A place for mathematical structure in the classroom

The teachers Cavanagh (2006) interviewed identified time pressure as one reason for not applying working mathematically in their teaching. They said that the content-driven curriculum did not encourage teachers to focus on these components of mathematical learning, and the need to prepare students for examinations was a barrier to incorporating activities that encouraged working mathematically. These are realistic and conspicuous reasons for mathematics teachers not to practice working mathematically in their day-to-day teaching. Similarly, teachers might not practice pedagogical approaches espousing mathematical structure because of these conditions.

Mathematical structure when identified in the working mathematically component of the NSW K-10 Mathematics (BOSTES, 2016) curriculum, complements the spirit of the NSW mathematics K-10 syllabus. It aims to promote an effective mathematical pedagogy that promotes students’ mathematical thinking.

**Professional development to increase awareness of mathematical structure**

Teachers’ awareness of mathematical structure is important as mathematical structure is a determinant of students’ mathematical understanding. Mathematical problems used by teachers during classroom instruction need to reflect their awareness, and appreciation of mathematical structure. Teachers need to transfer their mathematical structure awareness, as suggested by Mason et al. (2009), to their students. Students will then develop structural thinking through an awareness of mathematical relationships, and properties.

The effectiveness of mathematical structure when focusing on mathematical content and pedagogy was identified in a professional development program created by Vale et al. (2011). Teachers involved in this program reflected on their learnt experiences of understanding mathematical concepts, and appreciation of pedagogical knowledge. Their reflections indicated that they had deepened, and broadened their knowledge of teaching junior secondary mathematics, as well as developing their capacity to support students’ learning of mathematics. The researchers indicated that there is a need for further research in the area of teachers’ awareness of mathematical structure and how teachers could be encouraged to embed structure in their teaching.
practice. The development of professional development programs such as that implemented by Vale et al. (2011) can cultivate teachers’ awareness of mathematical structure, so they can implement strategies to deepen students’ mathematical understanding through structural thinking.

The need to identify how teachers can embed structure, through working mathematically, into their lessons then becomes a focus of mathematical pedagogy. The teacher’s awareness of mathematical structure, through mathematical structure, becomes a critical before it can be persuasively applied into the mathematics lesson.

**Mathematical structure and assessment practices**

A focus on mathematical structure means a focus on students’ understanding mathematical procedures and concepts through structural thinking. Understanding is the intrinsic rewards that motivates students, as Mason et al (2009) acknowledged, the knowing how, and why, motivates students to engage in mathematical learning, not the mark that appears on an assessment task. Boaler (2015) goes further to say that rigid assessment practices are detrimental to students learning. Timed tests usually require students to memorise facts and procedures to be reproduced in a controlled environment, and these can be in the form of quick quizzes of 5-10 minutes in the classroom, longer 30 – 60 minute end of topic tests, assessment tests, and 1.5–3 hour examinations that mimic the NSW Higher School Certificate (HSC). The expectations of students, parents, teachers, school executive, and the community in general is that by giving students smaller tests situations then they are slowly being prepared for the longer and more important examinations. This certainly is the case for student preparation to complete the HSC, but the growing number of students disengaging in mathematics by the manner that students are being prepared for this examination is proving to be a critical issue as to why this one of the reasons why this approach is failing to engage students, and motivating them to undertake higher levels of mathematics.

Granted that students need to be prepared for examinations at the higher levels of mathematics, but the over use of procedural teaching processes that junior secondary students are exposed to recall the content required has created what Mason et al.
(2009) called “memory burden”. Young students, simply cannot retain the amount of information required, they do not process it all in timed test situations. Attard (2010) identified that the transition from primary to secondary school was identified as causing anxiety in young students. Students are already anxious about mathematics and doing tests just supports this further. Nervousness interferes with ability to perform well in tests. There are many other issues that are not considered as influencing the students’ performance. Yet, it is the result on these timed assessments that students complete in junior secondary school determine their personal attitude, motivation, and confidence towards the subject. The result has seen fewer students enjoying mathematics learning, declining engagement in mathematics in the first years of secondary school leads to fewer students attempting higher levels of mathematics in senior years

When the teacher focuses on mathematical structure through the working mathematical component of the NSW mathematics syllabus, teachers are aware of all aspects of students mathematical thinking, and in particular the affective, or emotional, learning component. How a student feels about their ability, or their self-efficacy towards the subject, will impact on their motivation and interest in learning. Teaching towards procedures and memorisation of facts, places all mathematical learning into one category and fails to encourage students to think mathematically. When there is more emphasis on structural thinking students are able to think deeply about the mathematics learnt, and to apply their understanding learning test situations, with greater depth of understanding.

Mathematics teachers need to put mathematical structure through, the working mathematically aspect of the NSW mathematics curriculum, rather than a purely content focus, at the forefront of all their pedagogical practices in the classroom. By developing teaching strategies through these components, students will develop a deeper understanding of mathematics, increase their structural thinking skills, remember, and apply the mathematics to other situations beyond the timed test. Alternative teaching strategies, and assessment approaches that encourage students to think mathematically, communicate their ideas, listen to others, make alternative decisions and mistakes, and not be afraid to be wrong are components of structural thinking that should be encouraged in all mathematical learning. Mathematics that
Part 2 – A place for mathematical structure in the classroom

has one procedure, and one answer does not extend students learning and thinking. It simply restricts students thinking that there is only a right or wrong way, and if you know the right way then you are “good at maths” there is no alternative if the path chosen is the wrong way.
REFERENCES


Mark Gronow, January 2016
Part 2 – A place for mathematical structure in the classroom


Mark Gronow, January 2016