This is the final of three reports written to complete the Br John Taylor fellowship requirements. The theme of these reports is that of mathematical structure, this report focuses on how mathematical structure challenges current educational assessment and reporting procedures, and how it links with current research on mindsets. The author completed this research as a requirement for the Master of Education degree at Macquarie University under the supervision of Professor Joanne Mulligan.

**Focus on mathematical structure**

Mathematical structure is an alternative to the memorising of facts and procedures approach in mathematics teaching. Mathematics teachers can implement mathematical structure into their teaching through a greater awareness of how, and what students’ think when learning mathematics. The important perspectives of affective, and cognitive learning need to be considered. The affective learning domain, which is the emotional component of learning, influences the student’s cognitive understanding of the mathematical concepts, and his or her ability to reproduce these as learnt procedures. Mathematical structure is not content to be taught, it is implicit in the teaching and learning of mathematics to support students’ understanding, which includes student well-being. Feeling good about learning mathematics will improve engagement, motivation, and success. This discussion presented in this article identifies how the awareness of mathematical structure, through the working mathematically component of the NSW mathematics syllabus, will benefit students in the mathematics classroom, and post secondary school. The concept of growth mindsets is introduced to support mathematical structure as a crucial element for effective change to current pedagogical procedures in mathematics to improve student learning and engagement.

Mathematical structure and working mathematically are linked through familiar components of the working mathematically outcomes in the NSW K-10 mathematics syllabus (NSW Board of Studies, 2012). Working mathematically outcomes taught in isolation to the content, they are embedded in the content strands of syllabus: number and algebra, measurement and geometry, and statistics and probability like mathematical structure, they are the required for developing deeper understanding of mathematical concepts and procedures, and are essential in supporting students’ affective learning of mathematics. Unfortunately these outcomes, and their potential
to effect student engagement, are not fully realised. If they were, we would not be seeing the decline of student engagement in mathematics, and enrolment in advanced mathematics courses. In schools, mathematics programmes, scope and sequences, and units of work recognise working mathematically outcomes for administrative purposes, such as: assessment, reporting, and school inspections. Yet, are rarely acknowledged in the day-to-day teaching, or effectively addressed in assessment tasks. Cavanagh (2006) demonstrated this in teachers’ lack of awareness, and inclusion, of working mathematically in their every day teaching.

**Timed tests and homework**

The focus on timed test assessments, and procedural understanding has created students who are: disinterested, disengaged, unsuccessful and disliking mathematics (Boaler, 2015a). Many parents, and some mathematics teachers, will argue that the importance of a mark, the class/year average on a test, and a rank position in the group to show how the student is achieving in mathematics relative to others. The popular belief is that this is how mathematics has always been assessed, and should remain so, as everyone understands, and accepts it.

Conversely, against popular opinion, this is not the appropriate way to assess student mathematical understanding and ability. Additionally, it is not effective in developing engaged, motivated, and successful learners, as will be discussed further. Teachers set tests as a means to satisfying the rigid assessment procedures imposed by school policy. Tests allow teachers to answer parents’ questions about students’ achievement, work ethic, and participation in class. As well, tests promote a progression through the heavy content burden of the mathematics syllabus.

The traditional procedural approach to teaching mathematics to prepare students for tests and examinations is criticised by mathematical researchers, such as Boaler (2015a, 2015b) as failing our students. Students in their first year of secondary school are subject to unrealistic measurement of their mathematical ability in their performance on timed test. Tests do not measure mathematical ability, they are more like to test: effectiveness of home study, ability to manage time to complete all of the test, reading and comprehension skills to interpret the test questions, and especially, ability to overcome the anxiety under pressure. Attard (2010) identified that students
transitioning between primary and secondary school are alienated from mathematics because of these rigid assessment structures. Boaler (2015b) questions why we use these harsh assessment techniques when assessing student mathematical ability, as there is no other situation in adult work life that requires this type of performance assessment. Why then is it necessary to impose this regime upon our youngest, and most vulnerable? In most cases, the answer is because it is expected, timed tests separates those who can, from those who cannot, to reward those who can. This is has not been successful in developing a society where mathematical skills and interests are common, and success at mathematics a higher level levels are encouraged and rewarded.

As part of a procedural approach to teaching, students in junior secondary schools are expected to spend time in class, and at home of completing repetitive tasks that focus on memorising, rather than understanding, a procedure, in preparation for tests. The procedure has little importance in young peoples’ world, and is irrelevant beyond the content being taught at that time. Classwork followed by repeated tedious and meaningless homework does not develop motivated, and interested learners. Conner, Pope and Galloway (2009), found that homework created stress for the students, meant a loss of family time at home, and had little impact on achievement. There are few young students that are good at this procedural learning, particularly when their technological world allows them to achieve interesting, and relevant information quickly. Students see mathematics, in this light, as irrelevant, and boring (Brown, Brown, & Bibby, 2008).

**Twenty-first century learners**

Conrad Wolfram creator of wolfram alpha in his Ted talk [https://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_com puters](https://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_com puters) (Wolfram, 2010), talks about the importance of computers in learning mathematics. He is aware that computation skills are regarded as essential components of learning mathematics, but in today’s technological world they are not as important as they were 20 years ago. Computers, and calculators perform complex mathematical tasks more efficiently that humans can. Wolfram says that the skills needed to be developed in the mathematics classroom are not computational skills, but the ability to pose a question, to transfer knowledge from the real world to a
mathematical model, and to relay the model back to the real world to see if the original question is answered.

Boaler (2015a) noted that in 1970 the most valued employment skill was writing and computation, whereas in 1999, teamwork and problem solving are the top two employable skills, with computational skills ranked 12th. There is no need to teach students in 2015, the same skills that were expected in the 1970s. Today’s generation of learners’ are required to be structural thinkers who can work mathematically, with have the ability to problem solve, think critically, analyse, communicate, and reason. These are the essential skills of the future. Today’s students become bored very quickly with repetitive, and boring tasks that are represented by procedural learning. They need to be excited and engaged in mathematics to become successful learners of the future.

Mathematical mindsets
The creation of those who can, and those who cannot do mathematics is encouraged, and continually promoted through a procedural focus of memorising required for completing timed tests. To reinforce success on these tests, schools will rank students, further organising students into those who can, and cannot, then tell them how good or bad at mathematics they are by their placement in a ability level classes. Boeler, Wiliam, and Brown (2001) pointed out the harmful messages students receive when placed into ability level classes, both at the top and bottom.

There is a belief is that higher achieving students will be better grouped together in one class, and those less successful in their own class. Paek and Foster (2012) found that for many students that achieve good results on tests discontinue advanced mathematics courses when graded into higher level mathematics classes, as they have not developed a conceptual understanding as their successful experiences are as a result of memorising, through procedural learning. Within, the highest streamed classes there are students who feel inadequate, as they are not achieving at the same level as the better students. Marsh et al. (2014) researched the negative effect of putting capable students into selective schools, they found that this lowers personal self esteem, confidence and creates feelings of inadequacy because when compared to
others they feel they are never good enough, the result is a lower achievement compared to true potential.

Parents reinforce poor mathematics attitudes by telling their children that “I was not good at maths either”, as if it was a family trait. Eccles and Jacobs (1986) found that when mothers said to their daughters “I was not good at math at school” their daughter’s results declined. There is a belief that some people are gifted with a maths gene, or that not all people are capable of achieving at mathematics. This view has been contradicted by the work of Carol Dweck, a professor in psychology at Stanford University [http://mindsetonline.com/abouttheauthor/]. Dweck (2006) introduced the concept of mindsets, and has transformed the way we view intelligence. Intelligence is not fixed at birth, making some people born smarter than others. Dweck says that we can achieve our potential through the way we approach the situation. She describes people with growth mindsets, as those who believe that smartness increases with hard work, whereas people with fixed mindsets believe that while you can learn things, you cannot change your smartness. When people change their mindsets they start to believe they can learn at higher levels and change their learning pathways. In this Tedtalk [https://www.youtube.com/watch?v=hiiEeMN7vbQ] video, Dweck (2014), describes fixed and growth mindsets. Mindsets have been supported through foundations in neuroscience, and how brain plasticity allows for the growth of new neuron pathways. Boaler (2015b, p. 13) describes how brain activity increases when mistakes are made and neuron pathways are formed through mistakes, not by remembering correct responses. She asserts that there is a need to encourage students to think about the mathematics they do, and be prepared to make mistakes. Students need to be exhilarated in learning from the mistakes made, not penalised, as is done in tests. She is critical upon the procedural understanding approach to teaching mathematics as it only allows for correct answers, and discoursages mistake making.

Jo Boaler is a professor in mathematics education at Stanford University. Having taught mathematics in the UK and USA, from junior secondary to undergraduate levels, her focus is on improving mathematics education for all. She has researched, and written extensively on how students’ learn mathematics effectively, and holds a strong belief that mathematics is accessible to everyone. Her
website [www.youcubed.org](http://www.youcubed.org) is followed worldwide by teachers, parents and students. Her books: *What’s math got to do with it?* (Boaler, 2015a), and *Mathematical mindsets* Boaler (2015b) are changing the way we think about teaching mathematics.

Boaler (2015b) does not describe mathematical structure in the same manner as Mason, Stephens, and Watson (2009). Her focus is towards effective learning, and that by creating a learning environment that has this focus, then the intrinsic reward students need to engage in mathematical learning is their excitement in mathematical understanding. This approach supports Mason et al. (2009) view that intrinsic rewards of understanding mathematics are achieved through structural thinking.

Mathematics has been taught as a competition, often based on speed responses, that only a few students are acknowledged as capable of. Boaler (2015b) expresses the danger of schools, and some teachers, to identify these students as gifted in mathematics, and ignoring those who work hard to achieve. She points to research that shows that these students while doing well at school often do not perform, as well in adulthood. Where as, students who are persistent, work to overcome difficulties, and challenge themselves to do their best despite setbacks, are the ones who go on to achieve later in life. To perform well in year 7 and 8, students need to be good at timed tests, these students are good at memorising, but do not necessarily have the conceptual understanding that is required to do well in more advanced mathematics. Boaler (2015a) identifies these students, often high achieving girls, as having fixed mindsets, and lack the confidence to attempt more difficult mathematic for fear of failure. Where as the students with growth mindsets are not afraid of making mistakes, and learning from their mistakes. Students with the growth mindset will achieve at a greater level in year 11 and 12. Students who relied on memory to achieve do not achieve at the higher levels, as their methods of learning do not develop structural thinking skills. Students who worked towards understanding the concepts improved, and continue to improve, as they focused on conceptual understanding.

Boaler has created an online learning course for teachers, parents and students through [www.youcubed.org](http://www.youcubed.org) website. This video [https://www.youtube.com/watch?v=2jixf8gYdT0](https://www.youtube.com/watch?v=2jixf8gYdT0) (Jo Boaler, 2014a) is from
this online learning course for teachers. It explains mindsets in a mathematical classroom, and gives a practical example of how a middle school teacher took the problem of ‘one divided by a third’, to create a learning environment where students were open and engaged in a discussion, and applied structural thinking to solve the problem. To solve the problem were able to view alternative pathways, use visual displays to explain how they got their solution, and discuss why some methods were incorrect. Students were involved in structural thinking, and the teacher’s attention to mathematical structure on understanding of the division process, deepened the teacher’s pedagogical understanding of listening to the students explain their thinking, and developed a pedagogical process beyond a procedure. An algorithm to solve the problem, as would be taught in a procedural based classroom was acknowledged, but students were asked to ‘make sense’ of the question before the algorithm was introduced. The algorithm would still be used to solve other problems, but students understanding of division with fractions allows them to think deeply about the question being asked, and apply the knowledge learnt to the new situation. Students’ ability to transfer knowledge, make connections between previously learnt matter and new work, and the relationships between content are essential components of structural thinking. The teacher’s ability to structure a lesson that allows students to develop these structural thinking skills is essential for higher mathematical thinking.

**Including mathematical structure**

Mathematics teachers are not required to make radical changes to their current pedagogical practices to include mathematical structure into their teaching. Mason et al. (2009) states that mathematical structure cannot be taught, and it is the teachers’ awareness of this concept that develops students’ structural thinking. Vale et al. (2011) demonstrated how mathematical structure improved teachers understanding of mathematics, as well, made them aware of how students learn mathematics.

Mathematics teachers are aware of the need to teach concepts, and that students need to spend more time in consolidation of these concepts, and develop greater awareness of mathematics that makes sense to them. Boaler (2015a) calls it ‘making sense’, and once students make sense of the mathematics they are learning, then they develop a growth mindset that encourages engagement in mathematics. Once understanding of mathematical concepts occurs, then engagement will follow.
In the NSW K-10 mathematics syllabus we see the theme of working mathematically presented before the content strands. Addressing mathematical structure as an awareness of working mathematically through communicating, problem solving, reasoning, understanding, and fluency are easily associated with Boaler’s (2015b) work. In her book, Mathematical Mindsets (Boaler, 2015b,) presents the working mathematically components: reasoning, problem solving, and communication as essential requirements to mathematical learning.

Working mathematically becomes a focus of what teachers teach, not as an additional component to be included at the end of the content, or an additional question in an assessment task to satisfy school registration and accreditation requirements. Teachers need to teach towards mathematical structure in every lesson. Mathematical structure through working mathematically can be in the instructions given, the activities assigned, the discussion and collaboration between students.

**What needs to happen?**
The responsibility of improving student engagement in mathematics is not only the mathematics teacher’s responsibility. School executives must be forward thinking, and plan towards giving teachers the support they need to make the changes required to develop competent, and engaged learners of mathematics. The school community must be educated that mathematics ability is not about tests scores, class averages, and ranks. It is about independent and creative learners willing make mistakes, and feel confident to be challenged by their mistakes. Teachers need to be given the freedom to work their class at pace that is supportive of individual learner, common tests that suggest ‘a one size fits all’ is not benefitting the higher ability students, and simply demoralizes the students who need more time, and support.

Students, and teachers must be educated in mindsets, and encouraged to dismiss teaching dominantly to procedural understanding, which has proven unsuccessful, and does not develop a growth mindset to support challenging concepts, particularly for girls.

**The current situation**

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Our current focus on achieving band 6 scores in the HSC does not encourage students to take the higher levels of mathematics. Able students see that a good result in General Mathematics as leading towards a higher ATAR than a mediocre result in the higher mathematics course (Pitt, 2015). Rylands and Coady (2009) found this was detrimental to performance at university, as courses studied at school, and not university entrance scores determined success at university. Students who attempt the higher mathematics course do better at university. Their conceptual understanding of mathematics, and their structural thinking skills are at a higher level, this creates a mindset that encourages the challenge presented by difficult concepts. Whereas, the General Mathematics course relies on procedural understanding, without the development of deeper thinking skills.

As a result of the high stakes of HSC performance, schools are trending students towards General Mathematics to market their school with higher numbers of Band 6 results. A band 5 in Mathematics is substantially better than a band 6 in General Mathematics, but does not get recorded in newspapers or school newsletters. Schools should be encouraging students to work towards the more challenging courses, and be promoting the increasing number of students attempting such courses. Teachers actively encourage students to take lower level mathematics because it will make their class results look better when HSC analysis are required, and the school executive scrutinises student results against teacher performance.

Capable students are being encouraged to do less challenging mathematics courses to improve the schools marketability within the community. Schools need to resist these publicity ploys so students can develop to their full potential. Teachers should not feel threatened by students doing higher level of mathematics, they should be rewarded for encouraging, and supporting students who want to commit to the subject to meet the challenges of more difficult mathematical concepts. The process that is currently in place, is the continual dumbing down of students’ ability and rewarding of success that requires less effort.

**The mathematics learner we want to produce**
A future where young people will grow as adults willing to be forward thinkers, risk takers, collaborators, and problem solvers with growth mindsets capable of
continually learning, and accepting any challenge placed before them, will be the measure of a school’s success. Schools should not look at the current Year 12 HSC results, but look at where its students are five to ten years after they have left school. The HSC does not always identify successful adults; if schools marketing featured student success later in life we would see a marked difference in how we are educating our younger students. The need to focus on test scores would not be the priority, developing capable, and competent thinkers with growth mindsets would be the goal. In mathematics, to achieve this, the teachers need to introduce mathematical structure into their lessons through he working mathematically components of the NSW mathematics syllabus. School executives need to encourage teachers to pursue this direction through professional development and administrative changes to school assessment and reporting practices. Finally the school community needs to be informed that the goal of mathematics is not a mark on the page, but being engaged to want to work harder, challenge oneself, and make mistakes.

The following link from Jo Boaler (2014b), youcubed.org website, sums up the arguments presented here, www.youcubed.org/think-it-up.

REFERENCES


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